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Chaos Virtual Lab: Experimental Findings in the Chaotic Nonlinear Damped and Forced Oscillator

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Abstract

This report includes an updated list of experimental findings in the chaotic nonlinear damped and forced oscillator (NLDFO). Since this system follows the period bifurcation cascade route towards chaos, other systems following the same route may display the features reported in this paper. The findings here reported were obtained by means of computer simulation experiments with a Chaos Virtual Lab developed by the author of this report. Obviously, physical intuition dictates that some of the findings mentioned in this report are of general validity, existing no reason to be restricted to systems following the period bifurcation route towards chaos. Dual Maps of Return are introduced as a tool to attain a better understanding of the deployment of peaks and valleys of oscillations.

Keywords: Nonlinear dynamics, Chaos, Tomographies, State Space, NLDFO, Computational Physics.

Introduction

With the aim of analyzing the chaotic dynamics of some mathematical models from Physics by means of computer simulations, this author developed a chaos Virtual Lab. The mentioned virtual lab is interactive software that performs computer simulations in up to 40 million time steps; it is enabled to make extra-large graphs on scrollable frames much larger than a computer screen, which allows visualizing some interesting details that usually remained unknown due to the poor resolution of small size graphs. In this way, this author has extensively studied the chaotic dynamics of the nonlinear damped and forced oscillator (NLDFO) and, a list of experimental findings from that research is included in this document. However, these results are not necessarily restricted to those chaotic systems transitioning to chaos through a cascade of period bifurcations, like the NLDFO, some findings are of general validity and it may be expected to spring up in other chaotic systems too.

The NLDFO is the drosophila of physical chaos

Just like the fly of the fruit, (technically known as the Drosophila melanogaster) is the ideal organism for genetic research in Biology, the nonlinear damped and forced oscillator (NLDFO) is –on the experience of this researcher- the ideal mathematical model to study chaotic behavior in physical systems. The reason behind this declaration is that any person with enough physical intuition may conceive in his/her mind the image and movement of this oscillator and figure out the results of altering certain conditions in its motion. In this way by slightly changing some parameters in the computer simulation of the NLDFO, the investigator may be aware of what to more-or-less expect as a result of that perturbation. This allows the researcher to accept or reject certain outcomes as the simulation evolves and, do not go ahead completely blind. In this manner as the research progresses, some expectations concerning the behavior of the system are corroborated because they have logic and are interconnected with other results. When studying other vibrating systems like the Duffing or the Van der Pol oscillators, physical intuition is not so effective. Note that the described method is some kind of computational perturbation theory applied to the study of nonlinear dynamics.

Mathematical Model of the NLDFO

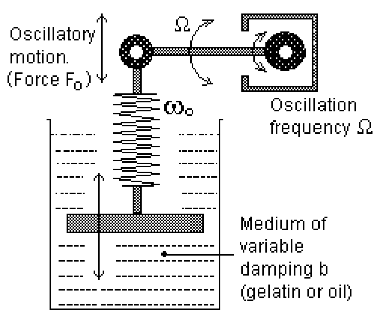

Figure 1. Forced oscillations in a medium of variable damping. The driving oscillating force is F_o and it is oscillating with frequency Ω.

As the immersed piece oscillates in gelatin or oil its motion changes the temperature of the medium thus varying its density and viscosity. Eventually there is a conflict between the applied and natural frequencies and the system starts to oscillate weirdly, traversing stages of declared chaos.

The investigation reported in this article is based on computer simulations of the oscillating system depicted in figure 1, which is a nonlinear damped and forced oscillator (NLDFO) [1-3]:

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_o^2 \, Sin \, x = F_o \, Sin \, \Omega t \qquad (1)$$

The system is a vibrator immersed in a medium of variable damping, such as oil or gelatin. In previous reports [4-9] this author has thoroughly studied the chaotic comportment of this system and, since all its chaotic events have the same behavior, one of these –the one identified as NLDFO(24)- was selected at random for this report.

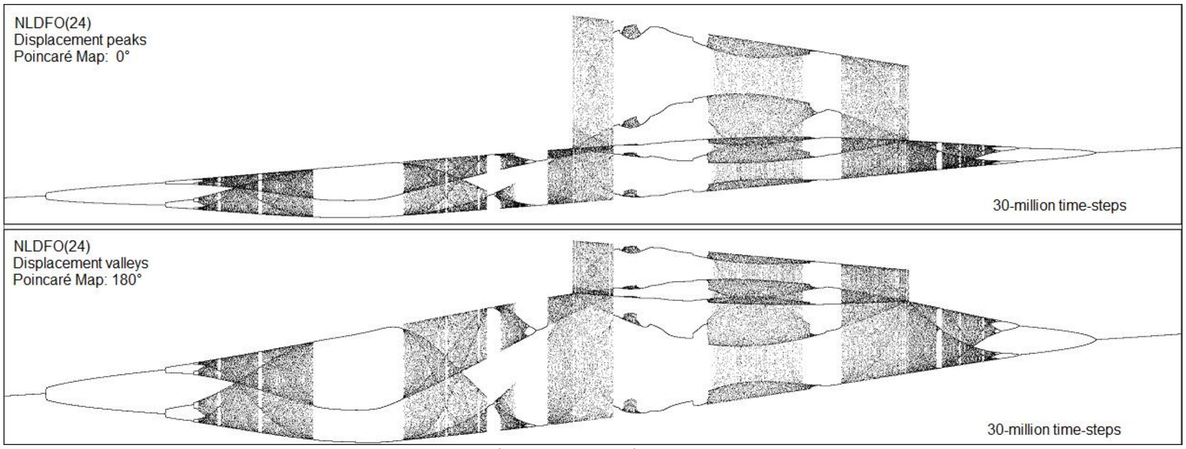

Figure 2. Longitudinal tomographies at 0° and at 180° of the state space of chaotic event NLDFO(24), showing the cascade of period bifurcations. The event runs along 30 million time steps. These two graphics show respectively the displacement extremes at both sides of the oscillation.

In figure 1, the continuous motion of the oscillator disturbs the temperature of the medium and in this way its density and viscosity varies while the oscillator vibrates. The oscillator, whose natural frequency is ω_o , is subject to an oscillatory driving motion force F_o whose frequency is Ω. As the immersed oscillator vibrates in gelatin or oil its motion changes the temperature of the medium thus varying its density and viscosity. Eventually there is a conflict between the applied and natural frequencies and the system starts to oscillate weirdly and chaos enters in scene.

As it can be appreciated, there are two competing frequencies in the system and there exists also an applied force and a variable damping. This constitutes the recipe [10,11] for a prone-to-chaos system. Note that the same effect may be attained without damping and with an increasing and oscillating applied force F_o.

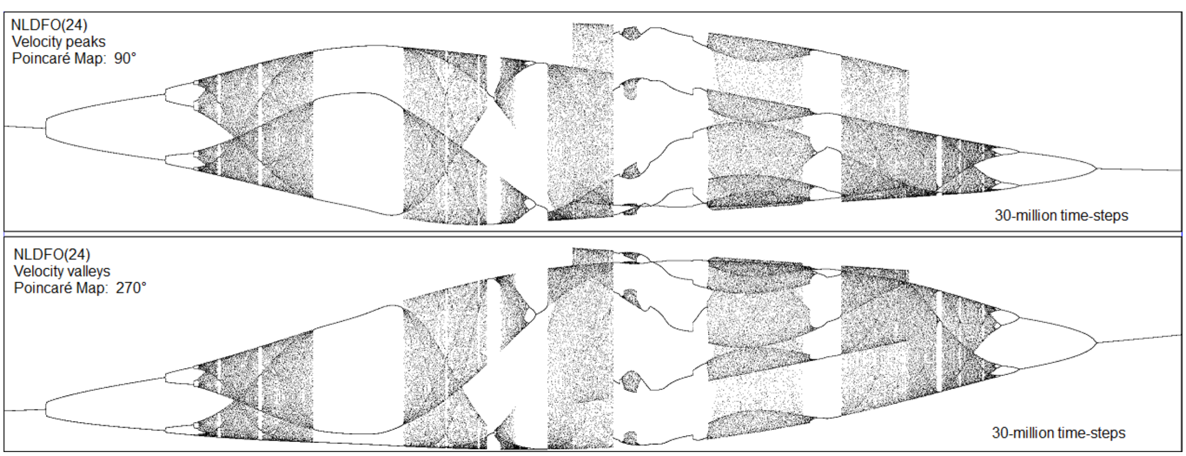

Figure 3. Longitudinal tomographies at 90° and at 270° of the state space of chaotic event NLDFO(24). These two graphs show the extremes of the velocity at both sides of the oscillation.

Maps of Return and Dual Maps of Return

A map of return [12] is the plotting of oscillation displacement extremes, one versus the previous, this is $x_n \; vs \; x_{n-1}$ and, when obtaining a map of return of either peaks or valleys, not much information is obtained, except maybe, that sometimes an interesting structure results [9,13,14], like a self-similar fractal, a straight line, a circle, etc. (see figure 4). Maybe the limited level of information that can be extracted from a map of return justifies its apparent occasional application. This researcher has obtained what may be called Dual Maps of Return, these are pairs of maps of return, like those shown in figures 5 and 6, where maps of return of both, peaks and valleys are shown simultaneously in a single graph and, using the same scale, thus allowing comparison between them, in this way, additional information concerning the unfolding of peaks and valleys of displacement may be gathered from those maps.

Figure 5 displays a dual map of return of the chaotic event NLDFO(24), whose bifurcation cascade is shown in figure 2, the upper cluster of points corresponds to the peaks of x(t) and the lower one is generated with the valleys of x(t) in that chaotic event. It can be seen that both return maps have self-similarity and have fractal structure; it may also be seen that the peaks of the oscillation have all positive values, while its valleys have positive and negative values, which means that during the chaotic episode the valleys of x(t) invade the positive region of the oscillations. Note that it is physically expected that the peaks of x(t) be restricted to the positive region of the oscillations and, the valleys of x(t) to the negative region. Since both maps are made with the same scale, it can also be concluded that the unfolding of the peaks of x(t) occupy a smaller region than that occupied by the valleys.

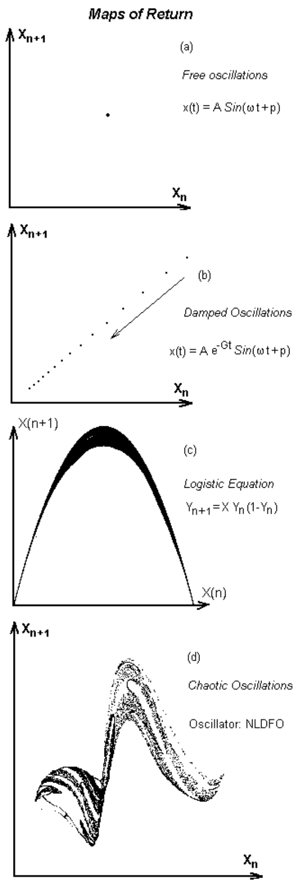

Figure 4. Maps of Return:
The plotting of the oscillation extremes
x_{n+1} *vs* x_n

(a) For free oscillations (constant amplitude).
(b) For a damped oscillator. (decreasing amplitudes)
(c) For the chaotic logistic equation.
(d) For one of the several chaotic events in the nonlinear damped and forced oscillator (NLDFO).

(a) Since all the extremes of the oscillation are the same, the map of return is a point. (b) Dots are aligned towards the origin, as amplitudes decrease continuously due to the damping. If the oscillations were stimulated, points would be projected away from the origin. (c) Dots are put up describing a parable. (d) Note that the return map of a chaotic event in the NLDFO is rather complex and it seems to have fractal structure and self-similarity.

Bear in mind that -as its name indicates- a map of return consists only of the extremes of the displacement, however, if deemed necessary it is also possible to develop maps of return of velocity and acceleration of any oscillation; in fact, the chaos virtual lab developed by this author allows generating them. Additionally, there is no need to develop maps of return from the very start to the very end of an oscillation; they might be constructed between predefined initial and final time-steps, indicated by the researcher, according to his needs.

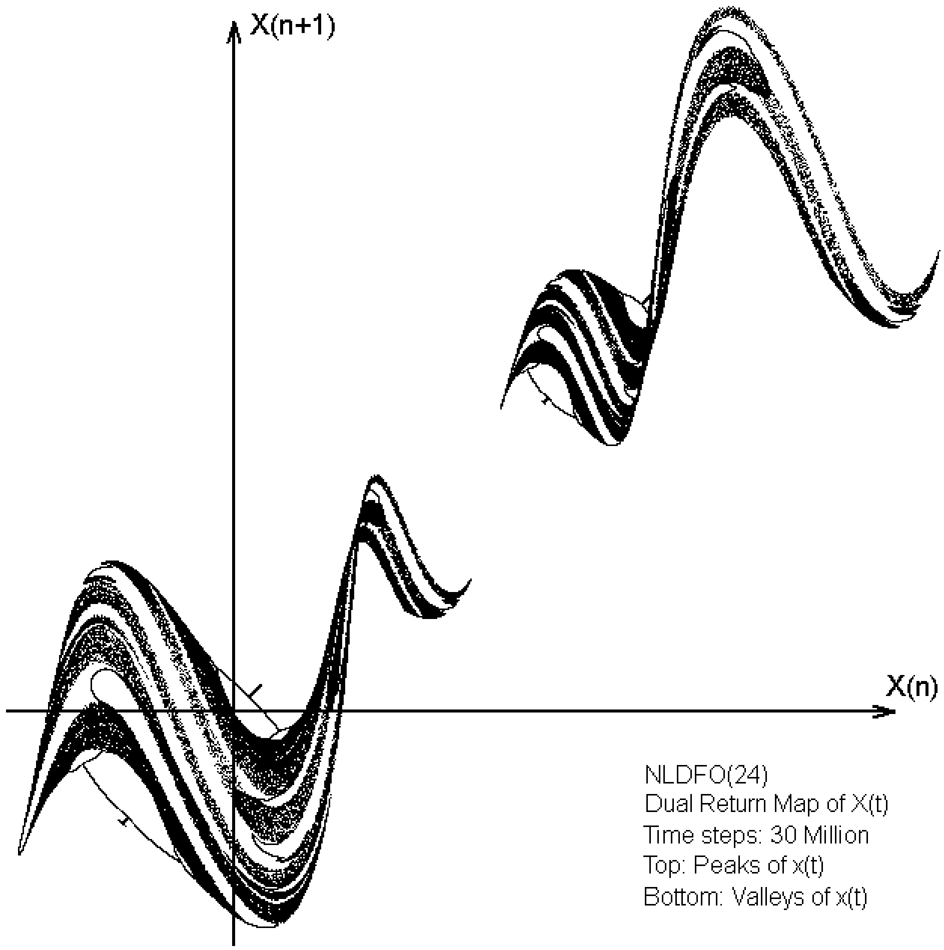

Figure 5. Dual Map of Return of Displacements obtained from the chaotic event #24 of the nonlinear damped and forced oscillator (NLDFO). As it can be seen the peaks of the displacement are always in the positive side of the oscillation, however the corresponding valleys are not restricted to the negative side –as it would expected- but eventually invade the positive side, which means that the oscillator rotates –though not necessarily in a uniform way- its equilibrium position. Observe that the maps of return have self-similarity and fractal structure. It can also be seen that in this event the deployment of peaks occupies roughly the same area as the valleys.

Figure 6 shows a dual map of return of velocities for chaotic event 24 in the NLDFO. Even though the name is weird because return is associated to displacement and not to velocity, this graph allows comparing the deployment of the extremes of the velocities. In the case shown in figure 6, it is obvious that the area occupied by the peaks of the velocity is similar to that occupied by the corresponding valleys; Notice that these maps of return seem to fold onto themselves, something not occurring with the displacements (figure 5). With the aim on achieving a deeper understanding into the apparent folding of these maps of return, it would be necessary to develop these maps between predefined initial and final points.

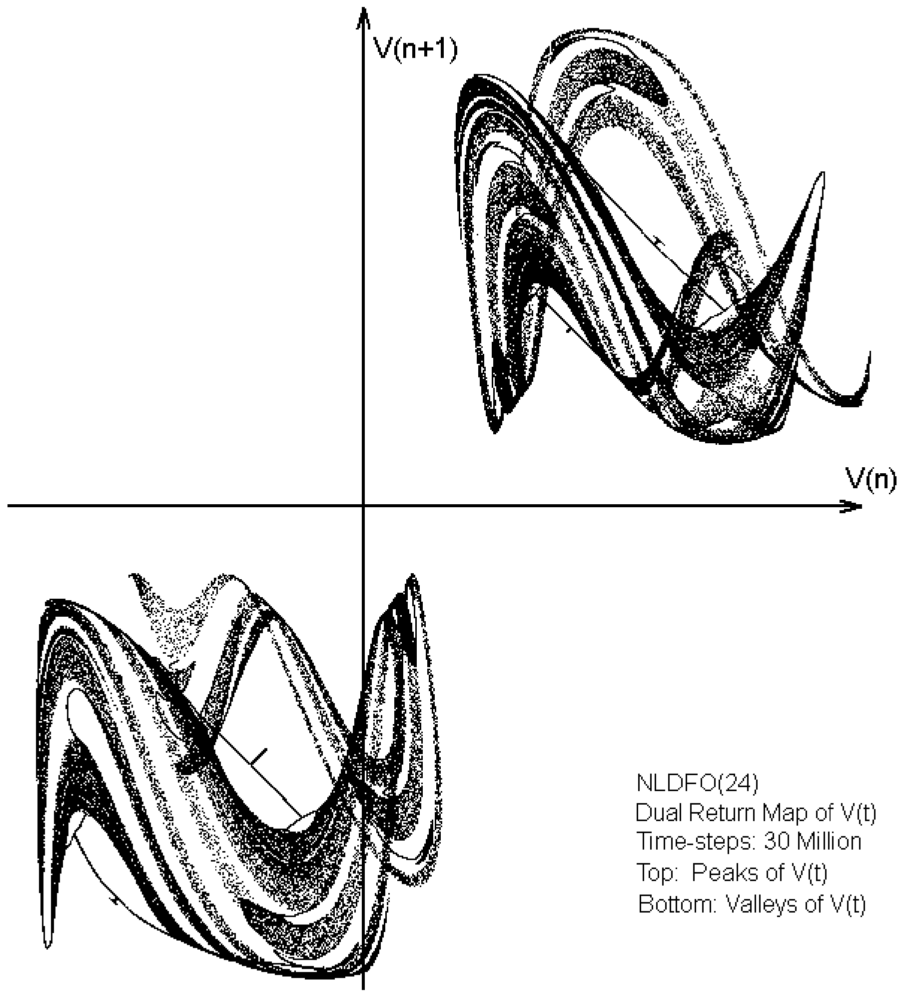

Figure 6. Dual Map of Return of Velocities. Even though the name "map of return" may not be appropriate in the case of the velocity, because return is associated to displacement, this plotting shows the distribution of peaks and valleys of the velocity during the chaotic event. At this moment this author ignores how to interpret this map as well as the information that may be obtained from it. Notice that these maps of return apparently fold onto themselves. It is also possible to construct a map of return of acceleration.

Experimental Findings from Chaos in the NLDFO

The following list includes results obtained by this researcher in this and in previous investigations [4-9, 13,14] executed on the chaotic dynamics of the nonlinear damped and forced oscillator (NLDFO); interested readers may seek more details in the mentioned references. Since the NLDFO transitions to chaos by means of a period bifurcation cascade, results mentioned here might be expected to appear in any system following the same route to chaos.

- As the chaotic oscillator vibrates it rotates its position of equilibrium during some lapses of time, however, these rotations are neither complete turns, nor in a fixed direction.
- There is a multiplicity of chaotic events in systems predisposed to to chaos.
- The multiple chaotic events that a system may exhibit are varied, this is, they are different among them.
- Chaotic events do not last forever, they are finite.
- Each chaotic event develops its particular state space; this implies different longitudinal Poincaré tomographies of this space, for different angles with its x-axis, which in turn implies different period bifurcation cascades.
- The extremes of the velocity during a chaotic event also generate bifurcation cascades. In these cascades the bifurcations and other details occur simultaneously with those at the extreme displacement (period) bifurcation cascades. This means that the extreme displacement bifurcation cascades and those of extreme velocities are similar and simultaneous.
- When the system evolves towards chaos, it does so by means of period bifurcation cascades. When the system abandons chaos, period bifurcation cascades are collapsed, not necessarily the same cascades.
- After a number of period bifurcations, chaos sets in; this appears as the region in the bifurcation cascade where it is impossible to foresee where the system will go next. An apparently complete disorder shows up in this stage of the oscillation.
- It has been mentioned that chaos appears and disappears between bifurcation cascades, but these cascades are not the same. In other words, there is no symmetry in the cascade before and after a chaotic stage.
- Short stages of relative quietness, where the system oscillates alternating among up to nine periods, may show up abruptly during chaotic events.
- Along the bifurcation cascades, zones with periods ranging from 1 to 9, have been observed, not necessarily in sequence. These regions may be interrupted by stages of furious chaos and also by "windows of (relative) tranquility".
- When the oscillator finishes a chaotic event, it returns to its regime of forced and damped oscillations.
- During a chaotic event, the extremes of the oscillations at both sides of the equilibrium position of the vibrator are not necessarily symmetrical. The same is true with the extremes of velocities.
- Evidences of serial chaos in the form of incipient bifurcations have been detected; these are hard to get because in order to investigate its existence, the chaos simulation must be extended over very long lapses of time after a chaotic event, but the incipient bifurcations suggest that serial chaos definitively exists.
- The Maps of Return of a chaotic event evidence self-similarity and, it seems to be they have a fractal structure.
- The higher the damping a chaotic oscillator is experiencing, this is, the higher the friction it is undergoing, the lower the furor of chaos and, consequently, the lower the Kolmogorov's entropy.

- The structure detected in the Poincaré's transversal tomographies of state space during a chaotic event, evidence that there is order and structure during chaos, even during its most furious stages.
- A complete chaotic event seems to consists of a series of attractors, which are completed with numbers of time-steps multiple of the attractor of period 1.
- When a chaotic event consists of a series of attractors, the transition from one to the next is smooth, i.e., the end of an attractor gradually becomes the beginning of the next.
- The number of (x,v,t) state-space points in a transversal tomography with N periods is a N-multiple of the number of those points in a single period.
- All attractors in chaotic events in the NLDFO seem to be individualized with a number of time-steps that is a multiple of the attractor with period-1. This suggests the existence of a single family of attractors in the chaotic events in the NLDFO.
- The dual maps of return of the chaotic NLDFO display self-similarity and fractal structure, these also show invasion of the positive region of oscillation by the valleys of x(t), this latter detail means that the oscillator rotates its equilibrium position during a chaotic event.

Figure 7 shows the period bifurcation cascades of some chaotic events detected in the nonlinear damped and forced oscillator (NLDFO). These bifurcation cascades are obtained as longitudinal tomographies [13,14] of state space at an angle of zero degrees with the x-axis in that space. These graphs evidence the finiteness and multiplicity of chaotic events, also the presence of windows of (relative) tranquility between stages of furious chaos is confirmed. It can be seen with the naked eye, that in the referred to as "windows of tranquility" the system oscillates alternating among up to nine periods.

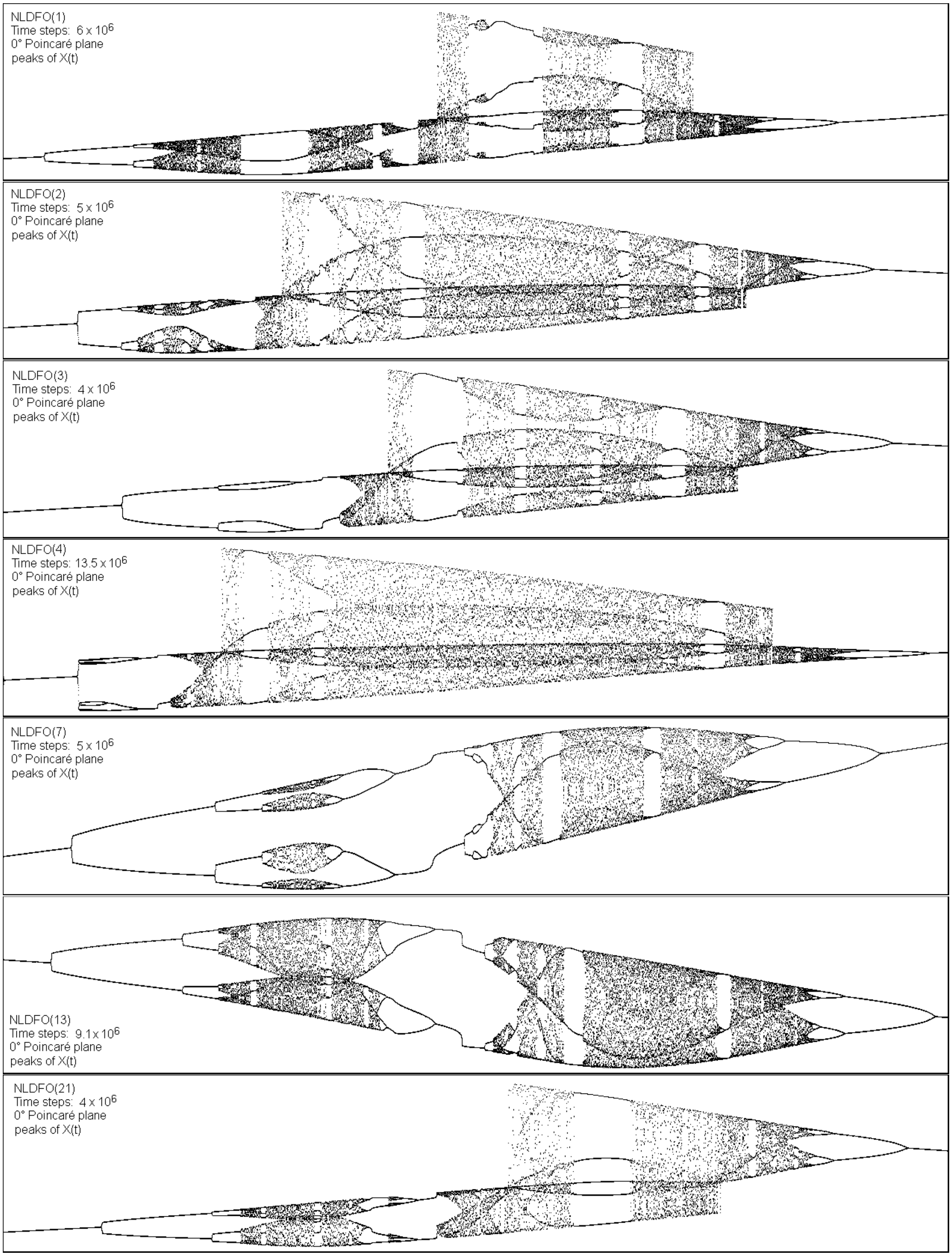

Figure 7. Period bifurcation cascades of some chaotic events detected in the NLDFO.

Conclusion

Based on the investigations carried out by this author on the dynamics of the chaotic nonlinear damped and forced oscillator (NLDFO), this paper reports an updated list of experimental findings in that system. Since the NLDFO follows the period bifurcation cascade route towards chaos, it is expected that other chaotic systems following the same route, show the same behavior. It is important to bear in mind that some of the reported findings are of general validity, meaning that they may spring up in other chaotic systems not necessarily following the same route towards chaos. Dual maps of return have been introduced as a tool to achieve insight into the unfolding of peaks and valleys of oscillations.

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