

EduVirtualLab Implementing the Logarithmic Decrement to Identify the Parameters of an Oscillation¹

Javier Montenegro Joo

VirtualDynamicsSoft: Science & Engineering Virtual Labs.
JMJ@VirtualDynamicsSoft.com ---- www.VirtualDynamicsSoft.com

Abstract

This paper reports the development of a university-level EduVirtualLab (educational virtual lab) that in order to identify the control parameters (viscosity, initial phase, maximum amplitude) of an oscillation, applies the technique of Logarithmic Decrement, LD. Once the user of the virtual lab supplies the input data of the oscillation, the module displays the corresponding curve of displacement vs time, $x(t)$, on computer screen. With the aim on obtaining reference theoretical results, the module automatically detects the extreme displacements (peaks and valleys) of the $x(t)$ curve and it applies the LD algorithm to these data to calculate the damping of the system and, from this the control parameters of the oscillation are identified. Next the module allows the user to click with the mouse on the peaks (or valleys) of $x(t)$ and, once the user has clicked 10 of these, the module computes the LD and from this, the experimental control parameters of the oscillation are obtained. The user may repeat this stage by clicking 10 valleys (or peaks) of $x(t)$. At the end of the simulation, the virtual lab displays both, theoretical as well as experimental results, so that the user can compare them. In this way, if the user has correctly clicked the extremes (peaks and/or valleys) of $x(t)$, his experimental results –as it is expected- are verified as being very close to the theoretical results calculated by the virtual lab. In order to automatically detect the extremes of $x(t)$ this module extracts Poincaré Maps, a technique from Chaos theory.

Keywords: Computational & Simulational Physics, Computers in Education, Virtual Experiments, EduVirtualLab, Damped oscillations, Logarithmic Decrement.

Introduction

The logarithmic decrement is a technique that by measuring the contraction of the oscillation amplitude of a damped system, estimates the damping that the system is experiencing. Due to this procedure, a more specific name for this technique would be the Logarithmic Decrement of the Amplitude. In this work, however, the original name will be respected.

The Logarithmic Decrement (LD), may be used to experimentally find the damping an oscillator is experiencing [1-4], then from this damping, the control parameters of the oscillation may be calculated. Here “control parameters” refers to the viscosity of the environment, angular frequency of the oscillator, its maximum amplitude and, its initial phase.

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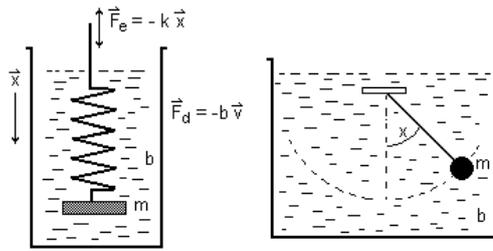


Figure 1. The oscillatory motion of spring and pendulum is attenuated by the viscosity of the medium, whose viscosity constant is b and its damping force is F_d . The elastic force of the oscillating spring is F_e .

This paper reports an enhancement of the virtual lab previously published by this author [1,2]. The algorithm described in the current paper shows how to experimentally extract two new parameters from the computer simulation, specifically the initial phase and the maximum amplitude of the oscillations. The extraction of these two new parameters adds evidence of the power of the LD technique to identify some parameters of the oscillation, reinforces the reasons to use computer simulation in some areas and, enhances the reported virtual lab.

In a previous paper [1] this author reported the creation of an EduVirtualLab, which after displaying on computer screen the curve of displacement versus time $x(t)$ based on user data, makes an automatic analysis of the curve to detect the damping the oscillator is experiencing, and from this, the frequency of the oscillations is computed. Next in [2] an improved and more advanced version of its predecessor is presented. In [2] the virtual lab becomes highly interactive and friendly, this time the user clicks with the mouse on computer screen on the peaks and valleys of the $x(t)$ curve and the module processes the information from these clicks and calculates the damping of the oscillations and from this, the frequency the oscillator is experiencing. Obviously this new version [2] of the Virtual Lab fosters learning by intense interaction of its user with the computer simulation.

The LD is defined as the natural logarithm of the ratio of any two successive extreme displacements in a damped oscillation. Obviously these two maximum amplitudes x_{n+1} and x_n are separated by a certain time t , so that:

$$\frac{x_{n+1}}{x_n} = e^{-\lambda t}, \quad t = t_{n+1} - t_n \tag{1}$$

where λ is a constant to be experimentally determined. The exponent is negative because in a damped system the amplitudes of the oscillations continuously shrink. In the case with no damping, if the amplitudes were constant, the exponent would be zero and, if the amplitudes of the oscillations would increase, the exponent would be positive.

Equation (1) is valid provided the oscillations are uniform; this is, as long as the distance between orbit turns in State Space keeps constant. By comparing with chaotic oscillators, in these the displacements are far from being uniform and the State Space is literally chaotic, in the most common sense of the word [5,6]. However, as it will be seen ahead, in order to automatically detect oscillation extremes, which are used to apply the LD, the Poincaré maps –a technique from Chaos Theory- may be used.

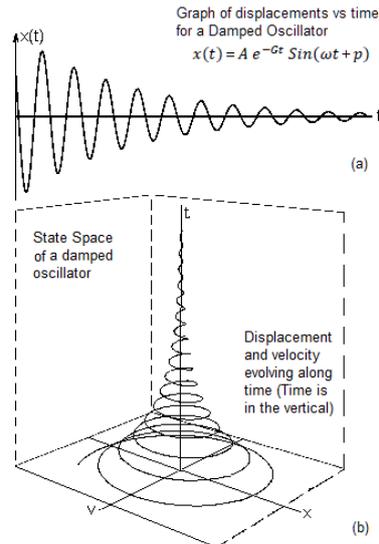


Figure 2. (a): time evolution $x(t)$ of the oscillation for a sub-critically damped oscillator.

(b): the corresponding State Space, this is the 3D-plotting of displacement and velocity versus time. It can be seen in both graphs that the amplitudes of oscillation decrease as time elapses, until finally they stop. Notice that the distance between turns in the state space orbit is constant, which means that the oscillations are uniform. In chaotic oscillations these turns are messy and unpredictable.

Damped oscillations

Damped oscillations (see Figs. 1 and 2) are characterized by the fact that the amplitudes of oscillation tend to reduce gradually as time elapses. Obviously, the higher the damping, the quicker the oscillations shrinkage. Depending on the relationship between the natural frequency of the oscillator and the applied damping, damped oscillations are classified as Critical, Subcritical and Supercritical. In this paper the subcritical case is dealt with, this case is also known as that of underdamped oscillations. Fig. 1 displays the two models of oscillating systems preferred by physicists, namely, the spring and the pendulum, and in this case, both are immersed in liquid, which provides the viscosity the oscillators are experiencing.

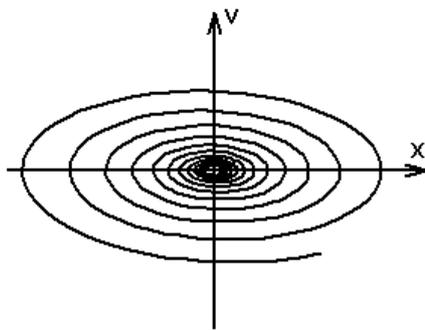


Figure 3. Projection of the (x,v,t) points of the 3D State Space over the XV -plane, for a damped oscillator.

This is a 2D version of the State Space. In this research the values of extreme displacements, this is, the points intersecting the x -axis in this plotting are used to feed eq. (10). The peaks of the displacement are the intersections of the curve with the x -axis at 0° and the valleys are those at 180° .

The differential equation of the underdamped oscillator

From elementary university physics it is known that the differential equation of motion of a system oscillating in presence of a damping [7-9] is

$$m \frac{d^2 \vec{x}}{dt^2} = -k \vec{x} - b \vec{v} \quad (2)$$

The first term on the right side of equation (2) is the reacting force of the spring (Hooke's Law) and the second term is the viscous damping, indicating that the faster the spring oscillates, the higher the resistance (minus sign) due to the viscosity b of the medium. Since the velocity is the first temporal derivative of the displacement, the scalar version of equation (2) is written as

$$\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

Here, two coefficients are identified:

$$\omega_0^2 = k/m \quad G = b/2m \quad (3)$$

where ω_o is the natural frequency of the oscillator, this is the frequency of the free oscillator, and G is the damping, which depends on the viscosity b of the medium.

After inserting ω_o and G in the differential equation it becomes

$$\frac{d^2x}{dt^2} + 2G \frac{dx}{dt} + \omega_o^2 x = 0 \quad (4)$$

In the underdamped case: $\omega_o > G$.

Since a damped oscillation tends to vanish along time, it can be assumed that the solution of equation (4) has the general form

$$x(t) = A e^{-Gt} \text{Sin}(\omega t + p) \quad (5)$$

Notice that in eq.(5) it is assumed that the system oscillates with a certain frequency ω , which as it will be seen ahead, is the frequency of the damped oscillations.

Equation (5) implies that the velocity of the oscillations –the first temporal derivative of $x(t)$ - is

$$\begin{aligned} v(t) &= \frac{dx}{dt} = A e^{-Gt} [\omega \text{Cos}(\omega t + p) - G \text{Sin}(\omega t + p)] \\ &= \omega A e^{-Gt} \text{Cos}(\omega t + p) - G x(t) \end{aligned} \quad (6)$$

And that the acceleration –the second temporal derivative of $x(t)$ - is

$$\begin{aligned} a(t) &= \frac{d^2x}{dt^2} = \omega A [-\omega e^{-Gt} \text{Sin}(\omega t + p) - G e^{-Gt} \text{Cos}(\omega t + p)] - G \frac{dx}{dt} \\ a(t) &= \frac{d^2x}{dt^2} = -\omega^2 A e^{-Gt} \text{Sin}(\omega t + p) - \omega G A e^{-Gt} \text{Cos}(\omega t + p) - G \frac{dx}{dt} \\ a(t) &= \frac{d^2x}{dt^2} = -\omega^2 x - G \left[\frac{dx}{dt} + Gx \right] - G \frac{dx}{dt} \\ a(t) &= \frac{d^2x}{dt^2} = -\omega^2 x - 2G \frac{dx}{dt} - G^2 x \end{aligned} \quad (7)$$

Finally replacing in the differential equation (4):

$$\begin{aligned} -\omega^2 x - 2G \frac{dx}{dt} - G^2 x + 2G \frac{dx}{dt} + \omega_o^2 x &= 0 \\ -\omega^2 - G^2 + \omega_o^2 &= 0 \end{aligned}$$

In this way:

$$\omega^2 = \omega_o^2 - G^2 \quad (8)$$

This result implies that (5) is a solution of (4) as long as condition (8) is satisfied.

Equation (8) supplies the connection between the natural frequency ω_o , and the frequency ω of the damped oscillations and the damping G experienced by the oscillator. Under these conditions the period T of the damped oscillations is $T = 2\pi/\omega$.

The displacement $x(t)$ and velocity $v(t)$, from (5) and (6) are used to construct the state space of the oscillations.

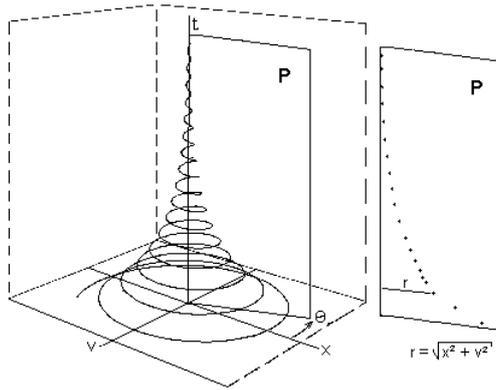


Figure 4. The Poincaré Plane P , may be seen as a tomographic cut along time in the 3D State Space. This plane P , is defined at some angle with the x -axis. The Poincaré Map is the set of all the intersections of the (x,v,t) curve with the plane P , at a predefined angle. It can be seen in this sketch that the peaks and valleys of the $x(t)$ curve are those tomographic cuts at 0° and at 180° , respectively.

The Logarithmic Decrement, LD

The LD [3,4,8] is based on the assumption that the shrinking in the maximum amplitude in a damped oscillation is given by

$$\frac{x_{n+1}}{x_n} = e^{-\lambda t} \quad (9)$$

where it is assumed that the shrinking of the orbit in state space is constant. In this expression the values of x_n and x_{n+1} are the extreme displacements of any two successive oscillations, these are the peaks (or the valleys) in the $x(t)$ plotting (see Fig. 2), or the points where the (x,v) curve cuts the x -axis in the 2D version of the state space at angles 0 and π with the x -axis, respectively (see Fig. 3). The extreme points x_n and x_{n+1} may be obtained by extracting Poincaré Maps of the State Space, as will be shown ahead.

From elementary oscillation physics, the elapsed time between any two successive displacement extremes is a period T , hence eq. (9) must be rewritten as

$$\frac{x_{n+1}}{x_n} = e^{-\lambda T}$$

Then after applying logarithms to both sides:

$$\lambda = -\frac{1}{T} \ln \left[\frac{x_{n+1}}{x_n} \right] \quad (10)$$

The period T in this equation is the period of the damped oscillations $T = 2\pi/\omega$. For the sake of accuracy, the value of λ in eq.(10) is obtained experimentally by averaging many instances of this equation.

On the other hand, eq.(5) may be applied to find the ratio of two successive maxima of the oscillation, which as it is known, are separated in time by a period, $t = T$:

$$\begin{cases} x_n = A e^{-G t} \text{Sin}(\omega t + p) & (11) \\ x_{n+1} = A e^{-G(t+T)} \text{Sin}(\omega(t+T) + p) & (12) \end{cases}$$

From eq.(12):

$$x_{n+1} = A e^{-G(t+T)} \text{Sin}(\omega t + p + \omega T)$$

but $\omega = 2\pi f = 2\pi/T$,

$$\begin{aligned} \text{then:} \\ x_{n+1} &= A e^{-G(t+T)} \text{Sin}(\omega t + p + 2\pi) \\ x_{n+1} &= A e^{-G(t+T)} \text{Sin}(\omega t + p) \end{aligned} \quad (13)$$

In this way from (11) and (13):

$$\frac{x_{n+1}}{x_n} = e^{-G T}$$

and taking logarithms to both sides of this equation:

$$G = -\frac{1}{T} \ln \left[\frac{x_{n+1}}{x_n} \right] \quad (14)$$

From equations (14), (10) and (8):

$$\lambda = G = \sqrt{\omega_0^2 - \omega^2} \quad (15)$$

Notice from equation (3), that the freshly found value of G may be used to compute the viscosity of the medium given by $b = 2mG$.

It is worthwhile mentioning at this point that equations (10) and (14) bring to memory the equation of the Lyapunov exponent from Chaos Theory. There the exponent is used to determine the intensity of chaos in a chaotic system [10,11], the main difference being that in order to compute the Lyapunov exponent two state-space orbits are implicated while in the case of the LD only one orbit is considered.

The Initial Phase and Maximum Amplitude of the Oscillations

It has been shown above that, the displacement and velocity of the oscillations are given respectively by eqs. (5) and (6):

$$\begin{cases} x(t) = Ae^{-Gt} \text{Sin}(\omega t + p) \\ v(t) = \omega Ae^{-Gt} \text{Cos}(\omega t + p) - Gx(t) \end{cases}$$

Then their initial values are

$$t = 0 : \begin{cases} x_o = A \text{Sin } p \\ v_o = \omega A \text{Cos } p - Gx_o \end{cases}$$

in this way:

$$\begin{cases} \text{Sin } p = x_o/A \\ \text{Cos } p = (v_o + Gx_o)/\omega A \end{cases}$$

Thus the initial phase of the oscillations is

$$\text{Tan } p = \frac{\text{Sin } p}{\text{Cos } p} = \frac{\omega x_o}{v_o + Gx_o} \Rightarrow p = \text{Tan}^{-1} \left[\frac{\omega x_o}{v_o + Gx_o} \right] \quad (16)$$

And the initial displacement (Amplitude) is:

$$\text{Sin}^2 p + \text{Cos}^2 p = 1 \Rightarrow A = \sqrt{x_o^2 + \left(\frac{v_o + Gx_o}{\omega} \right)^2} \quad (17)$$

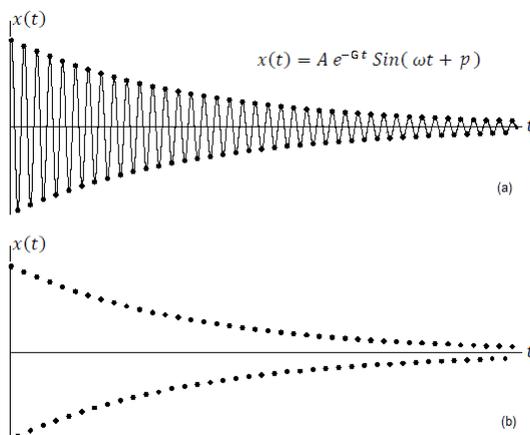


Figure 5. (a): Plotting of displacements versus time $x(t)$ for the damped oscillator. This plotting has been generated by the Virtual Lab being reported. Peaks and valleys have been highlighted (dots on curve extremes) after automatically extracting the Poincaré Maps at 0° and at 180° , respectively.

(b): The peaks and valleys to be used in the automatic stage of the virtual lab have been extracted.

Poincaré Maps: A technique from Chaos Theory

In studies of chaos based on computer simulation [12,13] the detection of peaks and valleys, this is, the extremes of the $x(t)$ curve, used in this experiment is tantamount to extracting the Poincaré Maps at 0° and at 180° respectively (see figure 4). Chaos theory does not work precisely with the logarithmic decrement, but the algorithm to extract the Poincaré Map may be used in the present work to extract the oscillation extremes. Hence, for someone who has developed the computer programs to extract Poincaré Maps in a chaotic system, the logarithmic decrement is not so unfamiliar after all.

The reported EduVirtualLab operates in two stages, namely "Automatic" and "Manual". The Poincaré maps technique is used only while in the Automatic Stage, to identify the peaks and valleys (extremes) of the $x(t)$ curve, which is generated as part of the operation of the virtual lab. In practical applications, when it is possible to identify these extremes by any other method, there is no need to extract any Poincaré map. This will be appreciated ahead in the "Manual Stage" of this work, where the extremes of $x(t)$ are identified by the experimenter, by clicking with the mouse on this curve, which is displayed on computer screen.

Some practical applications of the LD

The LD has been applied in civil engineering and in mechanical spectroscopy of solids:

Magalas [14,15] reports the creation of a new algorithm to compute the logarithmic decrement with high precision. This researcher introduces the Optimization in Multiple Intervals algorithm (OMI), which is recommended for measurements when the damping is rather high.

Butterworth et al [16] present an application of the logarithmic decrement in civil engineering, specifically to assess the dynamic response of structures (buildings) to vibrations due to earthquakes.

Majewski and Magalas [17] make use of the logarithmic decrement in the field of internal friction and mechanical spectroscopy of solids. In their paper these authors evaluate the application of the Hilbert transform to detect the envelope of damped oscillations with the aim on computing the LD.

Executing the experiment

The total process performed by the reported virtual lab may be appreciated in the flowchart in figure 6.

Automatic stage

Once the input data is entered the module automatically computes from eq.(3) theoretical parameters like the theoretical angular frequency ω_o of the free oscillator and the theoretical damping G of the medium. Additionally from eq. (8) the theoretical value of the angular frequency of the damped oscillations ω is computed, as well as the period T of these oscillations. Also at this stage the initial displacement x_o and initial velocity v_o are both calculated from eqs.(5) and (6), respectively.

Next the simulation of eq. (5) is executed during enough time for the oscillations to vanish. Notice that this is not indispensable at all, it would be enough running the experiment during a time equivalent to a few periods so that some values of x_n are available to evaluate eq. (10).

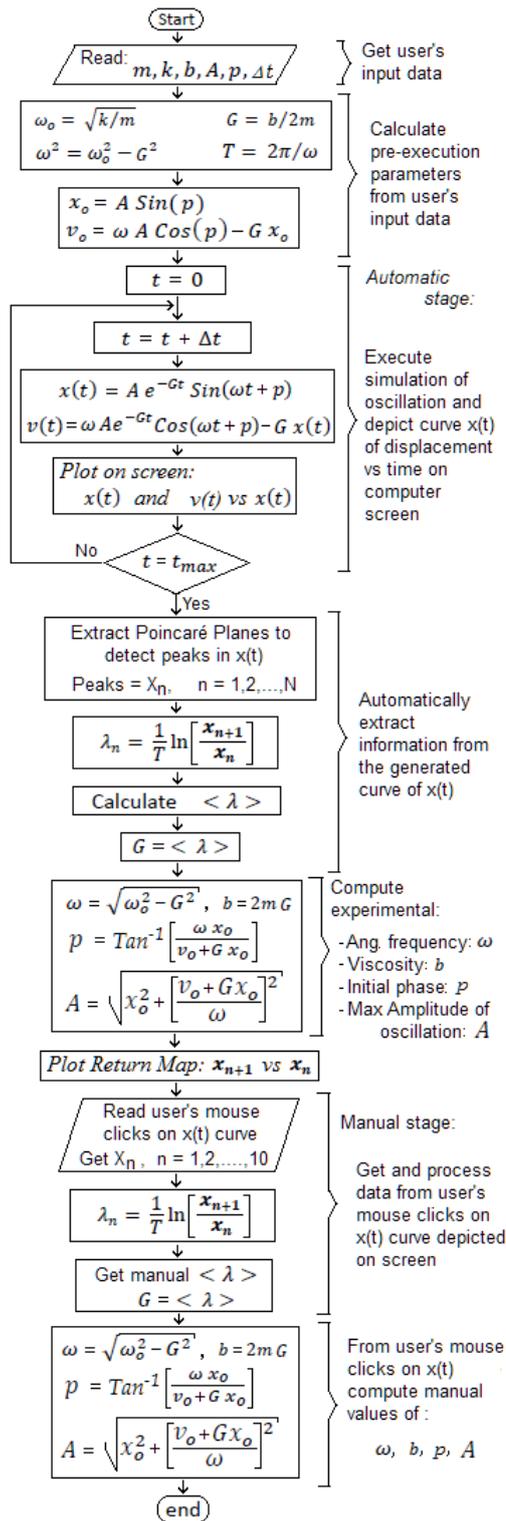


Figure 6. Flowchart of the process performed by the reported EduVirtualLab.

The virtual lab described in this paper operates in two stages, automatic and manual, in the former stage – which the user is not aware of- theoretical results based on automatic detection of the extremes of $x(t)$ are generated, while the latter stage is based on the user's interaction with the computer and generates experimental results. At the end of every simulation the user is enabled to compare theoretical with experimental results.

At the very beginning of an experiment, the virtual lab receives as input data the mass m of the oscillator, its elastic constant k as well as the viscosity b of the medium. Additionally the module reads the maximum amplitude “ A ” of the oscillations as well as its initial phase “ p ”. Also the time-interval Δt to be used in the simulation may be entered as part of the input data.

The maximum amplitude of oscillation, the initial phase and the time-interval may be entered by the user of the virtual lab so that she is enabled to appreciate the effects of changing them. Entering these data is not expressly fundamental for execution of the virtual experiment, because they have been included by default, however, larger values of “ A ” generate larger amplitudes of oscillation, and these are easier to visualize when displayed on computer screen.

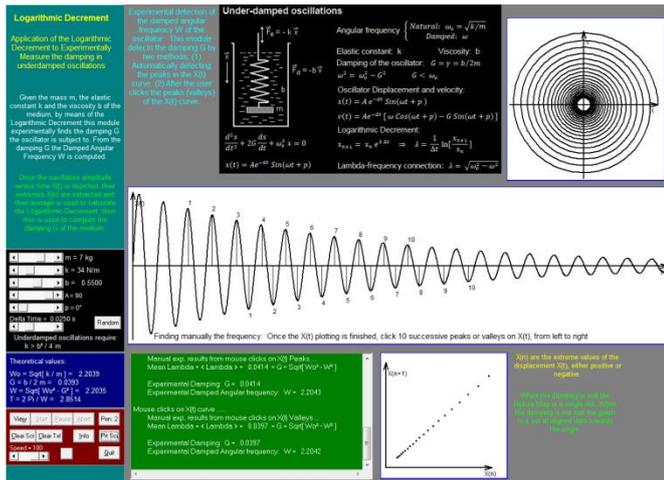


Figure 7. Screenshot of the Logarithmic Decrement EduirtualLab.

Besides the plotting of displacement vs time $x(t)$, the module shows a 2D projection of the State Space and a Return Map. The numbered (from 1 to 10) peaks and valleys on the $x(t)$ curve are those mouse-clicked by the user of the virtual lab.

<>< Virtual Lab: Logarithmic Decrement >>>

INPUT DATA

Mass $m = 2$ kg
 Elastic constant $k = 83$ N/m
 Viscosity $b = 0.3500$
 Amplitude $A = 84^\circ$
 Initial phase $p = 78^\circ = 1.3614$ rad
 Delta time = 0.0250 s

AUTOMATIC STAGE

Theoretical parameters:
 Damping of the oscillator $G = b / 2m = 0.0875$
 Natural Angular frequency $W_0 = 6.4420$ rad/s
 Damped angular frequency $W = 6.4415$ rad/s
 Damped Oscillations period $T = 0.9754$
 Initial displacement $X_0 = 82.1644$
 Initial Velocity $V_0 = 105.3079$
 N time-steps = 2500
 Elapsed time during oscillation = 59.900

Automatic calculation of Logarithmic Decrement

Damping: $G = 0.0875$
 Damped Angular frequency: $W = 6.4415$
 Amplitude $A = 83.9999^\circ$
 Initial Phase $p = 78.0002^\circ = 1.3614$ rad
 Viscosity $b = 2$ m $G = 0.35$

MANUAL STAGE

Results after mouse clicking the valleys of $X(t)$ curve ...

Damping: $G = 0.0851$
 Damped Angular frequency: $W = 6.4415$
 Amplitude $A = 83.9937^\circ$
 Initial phase $p = 78.0204^\circ$
 Viscosity $b = 2$ m $G = 0.3405$

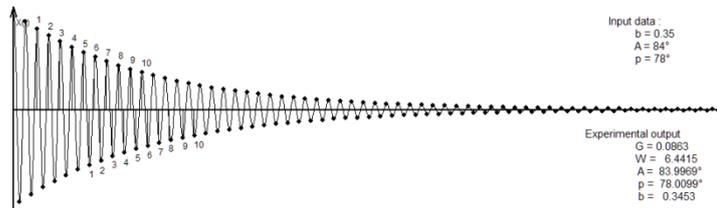
Results after mouse clicking the peaks of $X(t)$ curve ...

Damping: $G = 0.0876$
 Damped Angular frequency: $W = 6.4415$
 Amplitude $A = 84.0002^\circ$
 Initial phase $p = 77.9995^\circ$
 Viscosity $b = 2$ m $G = 0.3502$

Average results from clicking peaks and valleys

Damping: $G = 0.0863$
 Damped Ang. frequency: $W = 6.4415$
 Amplitude: $A = 83.9969^\circ$
 Initial phase: $p = 78.0099^\circ$
 Viscosity: $b = 0.3453$

Figure 8. Simulation report generated by the virtual lab during an experiment.



Above: The oscillation associated to the report displayed on the left side, whose input data are: $b=0.35$, $A = 84^\circ$, $p = 78^\circ$. After averaging the results from user's mouse-clicks on peaks and valleys, the output is: $G = 0.0863$, $W = 6.4415$, $A = 83.9969^\circ$, $p = 78.0099^\circ$, $b = 0.3453$.

Left: Once input data are entered by the user, theoretical results are obtained. Then the module automatically detects the oscillation extremes and it calculates the LD and from this, the control parameters of the oscillation. Finally after the user clicks with the mouse the extremes of the $x(t)$ curve, the value of the experimental control parameters are obtained. In this way the user can easily and quickly compare theoretical and experimental results.

Once the simulation starts, the curve of displacements vs time $x(t)$ as well as the state space are both gradually depicted on computer screen, as time goes by (see figures 3 and 5). Then by automatically extracting Poincaré maps at 0° and at 180° , the maximum displacements (peaks and valleys) of $x(t)$ are detected [5,6,11,12]. Next equation (10) is evaluated for every two successive maximum amplitudes of oscillation and the average λ is calculated and used together with eq. (15) to obtain the value of the damping G of the system and the frequency ω of the damped oscillations.

As it has been mentioned above, the virtual lab automatically detects all the peaks and valleys of the curve of displacement vs time, $x(t)$, however only a few peaks or valleys are necessary to assess eq.(10). This will be appreciated next in the manual stage of the experiment.

Manual Stage

In the manual stage, the virtual lab allows the user to identify with the naked eye 10 extremes (peaks) of the displacement vs time $x(t)$ curve displayed on screen. Right after the user clicks them with the mouse, the module computes the average logarithmic decrement λ and from this, the damping G the oscillator is subject to as well as the oscillation parameters. Usually these resulting data are close to their theoretical values calculated automatically right at the start of the experiment.

Next –if desired- the user may click the valleys of the $x(t)$ curve and obtain similar results. Once the user clicks with the mouse on computer screen 10 peaks (valleys) of $x(t)$ and the computer reports the corresponding experimental data, the user can additionally click 10 valleys (peaks) and, another set of experimental data are calculated. In order to improve statistics, the results obtained from mouse clicks on peaks are averaged with those from clicks on valleys.

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<<> Virtual Lab: Logarithmic Decrement <<>
INPUT DATA .....
Mass m = 7 kg
Elastic constant k = 87 N/m
Viscosity b = -0.5000
Amplitude A = 10°
Initial phase p = 68° = 1.1868 rad
Delta time = 0.0250 s

AUTOMATIC STAGE .....

Theoretical parameters:
Damping of the oscillator: G = -0.0357
Natural Angular frequency  $\omega_0$  = 3.5254 rad/s
Damped angular frequency  $\omega$  = 3.5252 rad/s
Damped Oscillations period T = 1.7823
Initial displacement  $x_0$  = 9.2718
Initial Velocity  $v_0$  = 13.5369
N time-steps = 2500
Elapsed time during oscillation = 62.525

Automatic calculation of Logarithmic Decrement
Damping: G = -0.0357
Damped Angular frequency:  $\omega$  = 3.5252
Amplitude A = 10.0000°
Initial Phase  $p$  = 67.9999° = 1.1868 rad
Viscosity b = 2 m G = -0.50

MANUAL STAGE .....

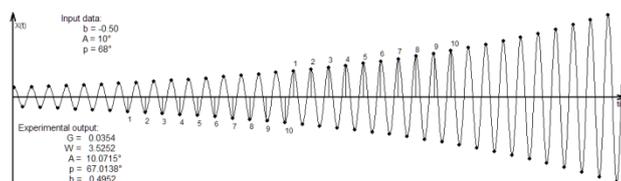
Results after mouse clicking the peaks of X(t) curve ....
Damping: G = 0.0355
Damped Angular frequency:  $\omega$  = 3.5252
Amplitude A = 10.0717°
Initial phase  $p$  = 67.0120°
Viscosity b = 2 m G = 0.4971

Results after mouse clicking the valleys of X(t) curve ....
Damping: G = 0.0352
Damped Angular frequency:  $\omega$  = 3.5252
Amplitude A = 10.0714°
Initial phase  $p$  = 67.0156°
Viscosity b = 2 m G = 0.4934

Average results from clicking peaks and valleys .....
Damping: G = 0.0354
Damped Ang. frequency:  $\omega$  = 3.5252
Amplitude: A = 10.0715°
Initial phase:  $p$  = 67.0138°
Viscosity: b = 0.4952

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Figure 9. Report of experiment generated by the virtual lab for the case of negative viscosity.



Above: The oscillation associated to the report displayed on the left side, whose input data are:
 $b = -0.50$, $A = 10$, $p = 68$.

After averaging the results from user's mouse-clicks on peaks and valleys, the output is:
 $G = 0.0354$, $\omega = 3.5252$, $A = 10.0715$, $p = 67.0138$,
 $b = 0.4952$

As it can be seen, with a few little adjustments-which are still to be implemented- the developed module may be successfully adapted to identify also the control parameters of oscillations with increasing amplitudes. In this case the name "Logarithmic Decrement, LD" would no longer be an adequate name for this application of the algorithm.

As usual the numbers from 1 to 10 on some extremes of the $x(t)$ curve are those mouse-clicked by the user of the virtual lab during the experimental stage.

A final checking verifies that the experimental values of oscillation control parameters are very close (if not equal) to their theoretical values.

Theoretically, there is no need to click 10 extremes (peaks and valleys) on the $x(t)$ curve, a pair of them would be enough. However using several extremes allows computing a more accurate result by taking their average.

Demonstrative execution of the EduVirtualLab

Damped Oscillations

In figure 8, a report generated by the virtual lab during an experiment is displayed. In this case the module has been fed with random input data. It may be seen that once input data are entered, theoretical results are computed. Then the module automatically detects the oscillation extremes and computes the LD and from this, the damping of the system and control parameters of the oscillation. In the manual stage the user clicks with the mouse the extremes (peaks) of the displacement $x(t)$ curve and the value of the damping is obtained. Then the process is repeated for the valleys of $x(t)$. It can be verified that the experimental control parameters are not so far from their theoretically calculated values.

Oscillations without damping.

When running a simulation with no viscosity the oscillations –as expected- do not shrink and the reported module finds the correct control parameters of the oscillation, which means that the LD technique operates –as expected- even in absence of damping.

Oscillations with increasing amplitudes

In figure 9, an oscillation with increasing amplitude, resulting from a “negative damping” is shown. This verifies that the Logarithmic Decrement algorithm successfully identifies also the control parameters in the case of oscillations with increasing amplitudes.

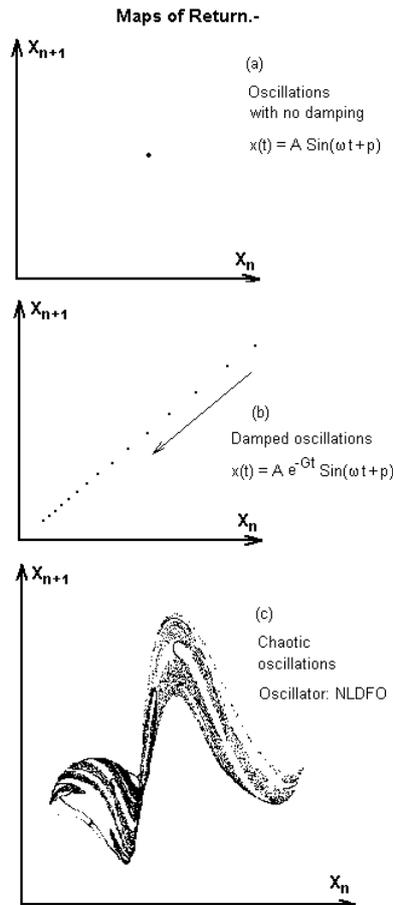


Figure 10. Maps of Return, the plotting of oscillation extremes x_{n+1} vs x_n (a) For oscillations with no damping (b) For the damped oscillator reported in this paper (c) For a chaotic oscillator, namely the nonlinear damped and forced oscillator. In case (b) the aligned dots go towards the origin, because due to the damping the oscillation amplitudes continuously decrease. In (c) the plotting is rather complex and seems to be self-similar (fractal structure).

In the case of increasing amplitudes of oscillation (“negative damping”), the aligned dots shown in (b) start near the origin and get away from it.

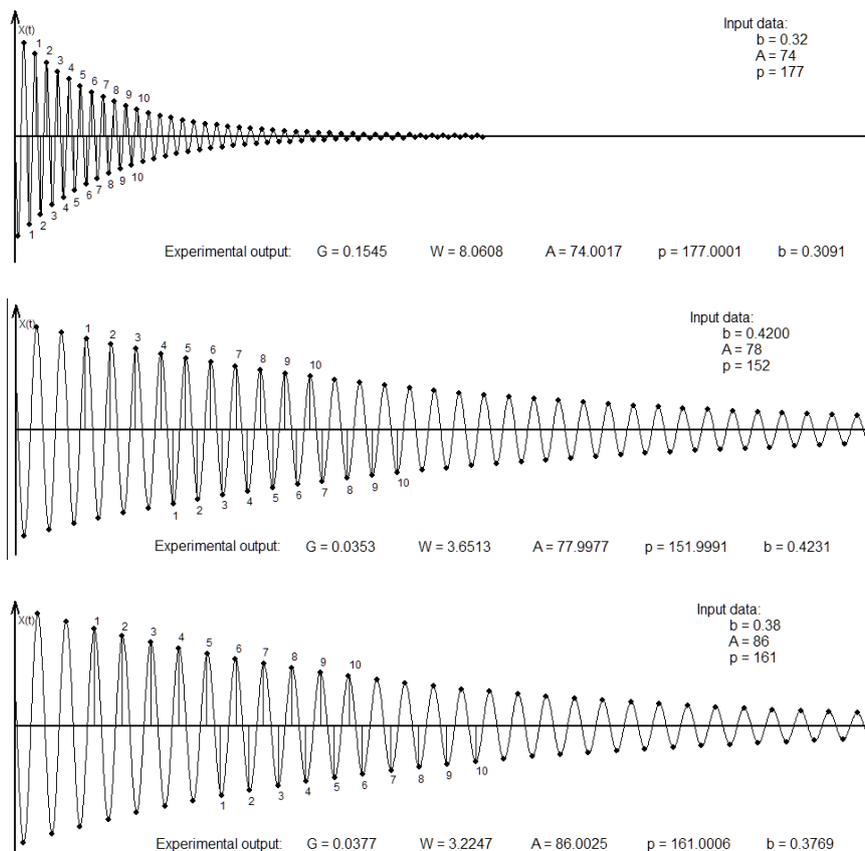
The Map of Return of displacements

The reported EduVirtualLab displays also the Map of Return for each experiment (see Fig. 10), which is the plotting of extreme displacements $x(n+1)$ vs $x(n)$. In the case of regular oscillations with no damping, the return map is a single dot, which means that all oscillation extremes are exactly the same, in the case of damped oscillations the graph is a set of aligned dots going towards the origin. It is instructive to compare these graphs with the return map of a chaotic oscillator, it results evident that in the latter case the plotting is not so easy to comprehend. Many times the return map of a chaotic oscillator seems to have the self-similarity typical of fractal structures. Notice that generalizing, the return map of velocity and acceleration may be used to study the evolution of these magnitudes, though in these two cases the name Return Map may be not so appropriate.

The three graphs shown in Fig. 10 are those obtained from the peaks of $x(t)$. In the first two cases the same plotting is obtained from the valleys of $x(t)$, however in the chaotic case shown in (c) the plotting obtained from the valleys may be similar but not equal to that obtained from the peaks of $x(t)$.

Some experimental results

Fig.(11) shows some experiments carried out with the EduVirtualLab described in this document. The graphs show some of the input data (viscosity, amplitude and initial phase) to run the simulation and the resulting curve of displacement vs time $x(t)$, which is displayed on computer screen. The numbered points from 1 to 10 are the mouse clicks of the user on $x(t)$. Each graph displays also the output parameters calculated after these clicks on $x(t)$. Obviously the more accurate the position of the click at the extremes of $x(t)$, the better will be the result.



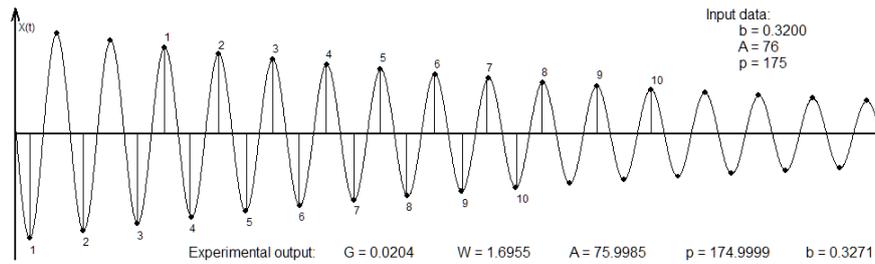


Figure 11. Some experimental results obtained with the reported virtual lab. The results under "Experimental output" are those obtained by averaging mouse clicks on peaks and valleys and, it can be seen that these are close to the corresponding input data.

Conclusions

This paper has reported the creation of an educational virtual lab (EduVirtualLab) to execute experiments on the application of the Logarithmic Decrement (LD) technique to identify the control parameters of an oscillation. It has been shown that the maps of Poincaré from Chaos Theory can be used to detect automatically the extremes (peaks and valleys) of the oscillation and then these are used to apply the LD technique.

Giving the mass of the oscillator as well as its elastic constant and the viscosity of the medium, as input data, the developed module experimentally finds the damping the oscillator is experiencing and, from this one, the oscillation frequency, the initial phase and the maximum amplitude of the oscillations (control parameters) are computed. It has been observed that even when there is no damping -this is zero viscosity- the algorithm reports the correct parameters of the oscillation. This paper shows also evidence that even for the case of increasing oscillation amplitudes ("negative damping"), the LD algorithm works successfully.

The simulation-based Virtual Lab reported in this paper sheds light on the fact that in everyday life, the control parameters of an oscillation may be identified by measuring –even manually- a few successive extremes of the oscillation amplitudes and applying the LD technique, not necessarily with computers.

The reported simulation module has been incorporated into the Physics Virtual Lab (PVL), created some time ago by this author and which is a collection of university-level intuitively-easy-to-use physics simulators [18,19]. The acronym "EduVirtualLab" means Educational Virtual Laboratory, it has been created by VirtualDynamics: Science & Engineering Virtual Labs and, is property of this institution.

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The Author

Javier Montenegro Joo studied Physics and Computer Science at San Marcos University (UNMSM, Lima, Perú), he has a Master of Science from Ohio University (Ohio, USA), where he also studied Physics and Computer Science. JMJ carried out doctoral studies in Computer Vision at Sao Paulo University (Sao Carlos, Brazil) specializing in Invariant Pattern Recognition. Since 1990 he works in virtual labs development for science and engineering (VirtualDynamicsSoft), currently the author also carries out research in Chaotic Dynamics and nonlinear systems.